Time Reversal Symmetry Breaking Effects in Resonant Nuclear Reactions*

H. Feshbach, M.S. Hussein†, A.K. Kerman

Center for Theoretical Physics
Laboratory for Nuclear Science and
Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139 USA

(CTP#2310 October, 1994)

To be published in: *Z. Phys. A*

Abstract

We incorporate time reversal symmetry breaking (TRSB) effects into the theory of compound nuclear reactions. We show that the only meaningful test of TRSB in the overlapping resonances regime is through the study of cross-section correlations. The effect is channel-dependent. In the isolated resonance regime, we employ K-matrix theory to show the impact of TRSB using the fact that when only one eigen-channel participates in populating and depopulating the compound resonance.

*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) under cooperative agreement DE-FC02-94ER40818

†Permanent address: Instituto de Física, Universidade de São Paulo, C.P. 20516, São Paulo, S.P. Brazil. Supported partly by FAPESP (Brazil)

I. INTRODUCTION

The breaking of CP symmetry observed for neutral kaons [1] implies, because of CTP invariance, time reversal symmetry breaking (TRSB), which has never been demonstrated experimentally. Because of the results of tests looking for TRSB effects, expectations are that they will be small. We believe, however, that the amplification which occurs in low energy neutron reactions may make them visible — and in any event will provide limits. [2] [3]. This feasibility is suggested by the recent observation of the violation of parity conservation using neutron resonances [4]. In this note we present a summary of the reaction theory needed to extract time symmetry breaking from both resonance and energy averaged experiments. In the following, we will discuss first, overlapping resonances and, in the next section, the isolated resonance regime.

II. OVERLAPPING RESONANCES

We make use of the methods employed by Kawai, Kerman and McVoy (KKM) [5] to obtain the average fluctuation cross-section. Because of time symmetry breaking the S matrix is not symmetric, $S_{ab} \neq S_{ba}$. In the case of neutron resonances, S has a form generalized from KKM, i.e.

$$S_{ab} = \bar{S}_{ab} - i \sum_q \frac{g_{qa} \check{g}_{qb}}{E - E_q} = \bar{S}_{ab} + S_{ab}^{fl} \tag{2.1}$$

In this equation $\bar{S}$ is the optical model S matrix while the g's are given by

$$g_{qa} = \sqrt{2\pi} \langle \tilde{q} | H_{QP} | \psi_a^{(+)} \rangle \equiv g_{qa}^0 + \Delta g_{qa} \tag{2.2}$$

$$\check{g}_{qa} = \sqrt{2\pi} \langle \psi_a^{(-)} | H_{PQ} | q \rangle \equiv g_{qa}^0 - \Delta g_{qa} \tag{2.3}$$

where Δg represents the effect of the breaking of time reversal symmetry on the fluctuations. Under time reversal invariance $\Delta g = 0$. Notice that both g^0 and Δg are complex.

The complex energies E_q, and the vectors $|q>$ and $<\tilde{q}|$ are solutions of the Schrödinger equations:

$$\left(E - H_{QQ} - H_{QP} \frac{1}{E^+ - H_{PP}} H_{PQ} \right) |q\rangle = E_q |q\rangle \tag{2.4}$$

and

$$\left(E - H_{QQ}^+ - \left(H_{QP} \frac{1}{E^+ - H_{PP}} H_{PQ} \right)^+ \right) |\tilde{q}\rangle = E_q^* |\tilde{q}\rangle \tag{2.5}$$

Our notation is that of Ch. III of the "Theory of Nuclear Reactions" [6]. The symmetry breaking interaction is present in H_{QQ}, H_{QP}, H_{PQ} and H_{PP} since

$$H^T \neq H$$

where H^T is the time reverse of H. As one can see from Eq. (2.2) and Eq. (2.3), symmetry breaking effects in nuclear reactions will have their source in H_{QP}, H_{PQ} and in $|q\rangle$. In

particular we note $< r|q > \neq < \tilde{q}|r >^*$. The resonance energies E_q, even though they have been very slightly changed by TRSB, will be averaged and will not affect the final result.

Assuming that g^0 and Δg are independently random. The energy averaged fluctuation cross-section is given by

$$\langle \sigma_{ab}^{fl} \rangle = \langle S_{ab}^{fl} S_{ab}^{fl*} \rangle \ . \tag{2.6}$$

which, with the usual random phase assumption, gives

$$\langle \sigma_{ab}^{fl} \rangle = x_0^2 \langle g_{qa} \breve{g}_{qb} g_{qa}^* \breve{g}_{qb}^* \rangle_q \tag{2.7}$$

We now introduce the following quantities obtained by applying the KKM analysis [7]:

$$X_{ab} = x_0 \langle g_{qa}^0 g_{qb}^{0*} \rangle_q \ , \quad x_{ab} = x_0 \langle \Delta g_{qa} \Delta g_{qb}^* \rangle_q \tag{2.8}$$

In Eq. (2.8) $x_0 = \sqrt{\frac{2\pi}{\Gamma D}}$, where Γ and $D$are the average width and spacing of the resonance. This constant will drop out in our final expressions. The average on the RHS of Eqs. (2.7) and (2.8) is carried out over the compound nuclear states as indicated by the subscript q. In terms of these quantities, the energy averaged fluctuation cross-section is given by

$$\langle \sigma_{ab}^{fl} \rangle = (X + x)_{aa} (X + x)_{bb} + (X - x)_{ab}^2 \tag{2.9}$$

where pair correlations among the g_{ga}'s and the random independence of g^0 and Δg have been assumed in performing the average in Eq. (2.7). It is assumed that the number of levels is large and the number of exit channels is large.

As in the KKM example the above quantities can be related to the transmission coefficients which are given by the optical model

$$T_{ab} \equiv \delta_{ab} - (\bar{S}\bar{S}^+)_{ab} = \left\langle (S^{fl} S^{fl+})_{ab} \right\rangle \tag{2.10}$$

where $\bar{S}$ is the optical model S matrix. The last expression can be related to X and x:

$$T_{ab} = (X + x)_{ab} Tr(X + x) + \left((X - x)^2 \right)_{ab} \tag{2.11}$$

We can now formulate the consequences of the above analysis. First note that according to Eq. (2.9), $\langle \sigma_{ab}^{fl} \rangle = \langle \sigma_{ba}^{fl} \rangle$. An early experimental test of this was reported in [3]. Thus it is not possible to detect time reversal symmetry breaking by comparing the energy averaged cross sections for $a \to b$ with that for $b \to a$. Detailed balance holds in the presence of symmetry breaking. To observe symmetry breaking we have to analyze the appropriate cross-section correlation function C_{ab}

$$C_{ab} = \frac{\langle \sigma_{ab}^{fl} \sigma_{ba}^{fl} \rangle - \langle \sigma_{ab}^{fl} \sigma_{ab}^{fl} \rangle}{\langle \sigma_{ab}^{fl} \rangle \langle \sigma_{ab}^{fl} \rangle} \tag{2.12}$$

It can be shown using the pair correlation assumption that to first order in N^{-1}, where N is the number of open channels,

$$C_{ab} = \frac{\langle \tilde{\sigma}_{ab}^{fl} \rangle^2 - \langle \sigma_{ab}^{fl} \rangle^2}{\langle \sigma_{ba}^{fl} \rangle^2} \tag{2.13}$$

where we have introduced the pseudo-fluctuation cross-section

$$\begin{aligned}\langle\tilde{\sigma}^{fl}_{ab}\rangle &= \langle S_{ab}S^*_{ba}\rangle \\ &= (X-x)_{aa}(X-x)_{bb} + (X+x)^2_{ab}\end{aligned} \tag{2.14}$$

From Eq. (2.9), Eq. (2.14) and Eq. (2.11), neglecting the non-diagonal terms ($X_{ab} = 0 = x_{ab}$), one finds, to leading order in the TRSB matrix element, the following form for C_{ab}:

$$C_{ab} \simeq -4\frac{x_{aa}X_{bb} + X_{aa}x_{bb}}{X_{aa}X_{bb}}\,, \tag{2.15}$$

when written in terms of the transmission coefficient, we find

$$C_{ab} \approx -4\left(\frac{t_{aa}}{\overset{\circ}{T}_{aa}} + \frac{t_{bb}}{\overset{\circ}{T}_{bb}}\right)\,, \tag{2.16}$$

where $\overset{\circ}{T}_{aa}$ is the optical transmission matrix element in channel a without TRSB, and t_{aa} is just the difference $(T_{aa} - \overset{\circ}{T}_{aa})$. We should mention here that the variance defined by $V_{ab} = \langle\sigma^{fl}_{ab}\rangle^2 - \langle\sigma^{fl}_{ab}\rangle^2$ has been carefully investigated by Kerman and Sevgen [8]. These authors pointed out that V_{ab} depends explicitly on Γ/D. Therefore, to study TRSB, one should avoid using V_{ab} and instead use C_{ab} where the Γ/D factors cancel out.

Equation (2.16) clearly shows that C_{ab} depends on the channels. This is in contrast to the result of Refs. [9], [10] and [11]. In particular Ref. [10] calculated $|R_{ab}|^2 \equiv \frac{\left[\langle\sigma^{fl}_{ab}\sigma^{fl}_{ba}\rangle - \langle\sigma^{fl}_{ab}\rangle^2\right]}{\langle\sigma^{fl}_{ab}\rangle^2}$ and claimed that it does not depend on a and b. The reason for this is that the authors of [9–11] consider TRSB to be entirely in H_{QQ} and do not consider its effect on H_{PQ}. Thus, if treated as purely internal mixing, the TRSB is channel independent, both in R_{ab} and C_{ab}.

The use of symmetry breaking one-body potentials to treat energy-averaged observables has already been proven successful in the case of parity non-conservation [12], [13]. An optical model description of TRSB [14], following the lines of [12], has been presented recently. It would be profitable to calculate C_{ab}, Eq. (2.16), using such one-body models of TRSB.

III. ISOLATED RESONANCES

We turn now to the case of isolated resonances. This situation is usually encountered at neutron energies in the electron volt region. The parity non-conservation experiment of Ref. [4] was performed under these conditions. The study of TRSB in the isolated resonance regime has been discussed recently [15,16]. Here we present a different point of view concerning this matter. It is convenient for the discussion to use the K-Matrix, which at a given isolated resonance, we write, in the presence of TRSB, as

$$K^q_{cc'} = \frac{1}{2\pi}\frac{\gamma_{qc}\gamma^*_{qc'}}{E - \epsilon_q}\,, \tag{3.1}$$

K^q is hermitian but neither not real, nor is it symmetric. Note the ϵ_q in (3.1) is the real energy of the compound level, q. The T-matrix is obtained from the K-matrix through

$$T^q_{cc'} = \frac{1}{2\pi}\frac{\gamma_{qc}\gamma^*_{qc'}}{E-\epsilon_q+i\Gamma_q/2}\,, \qquad \text{with } \Gamma_q = 2\pi\sum_c |\gamma_{qc}|^2 + \Gamma^\gamma_q\,, \tag{3.2}$$

with Γ^γ_q being the radiative decay width of resonance q, which is the dominant piece of Γ_q. The above form of $T^q_{cc'}$ establishes the link with the discussion at the beginning of the paper, *i.e.* for the present case $g^0_{qc} = Re\gamma_{qc} = \mathring{\gamma}$ and $\Delta g_{qc} = iIm\gamma_{qc} \equiv i\gamma^w_{qc}$. Going back to Eq. (3.1), we introduce the eigenchannels that diagonalize $K^q_{cc'}$, by the requirement $\gamma_{qc}\sum_{c'}\gamma^*_{qc'}f_{c'} = \lambda f_c$, which is solved by $\lambda = \sum_{c'}|\gamma_{qc'}|^2$ and $f_c = \gamma_{qc}$. All other solutions have $\lambda = 0$. Thus there is only one physical eigenchannel for each level q. We thus write for K^q in operator form

$$K^q = \frac{|\gamma_q|^2}{E-\epsilon_q}|\hat{\gamma}_q\rangle\langle\hat{\gamma}_q| \tag{3.3}$$

where $|\hat{\gamma}\rangle$ is a unit vector with components $\gamma_{qc}/\sqrt{\sum_c|\gamma_{qc}|^2}$. Note that this eigenchannel also diagonalizes T^q.

$$T^q = \frac{|\gamma_q|^2}{E-\epsilon_q+i\Gamma_q/2}|\hat{\gamma}_q\rangle\langle\hat{\gamma}_q|\ . \tag{3.4}$$

If we represent the TRSB measurement operator by θ_T, then the difference in total cross-sections with two different neutron helicities and a polarized nuclear target is

$$\Delta\sigma_q = \frac{2\pi}{k^2}Im\frac{1}{E-\epsilon_q+i\frac{\Gamma_q}{2}}\sum_{c',c}\langle\gamma_{qc}|\theta_T|\gamma_{qc'}\rangle \tag{3.5}$$

where c denotes the entrance channel, and c' the channel that θ_T couples. Since θ_T is by definition Hermitian and antisymmetric, $\langle\gamma_{qc}|\theta_T|\gamma_{qc'}\rangle$ must be purely imaginary. Accordingly, we have at the q-th resonance,

$$\Delta\sigma_q = \frac{2\pi}{k^2}\frac{2}{\Gamma_q}\sum_{c,c'}\left[\gamma^*_{qc'}\gamma_{qc} - \gamma_{qc'}\gamma^*_{qc}\right](\theta_T)_{cc'}\ . \tag{3.6}$$

As noted above we have $\gamma_{qc} = \mathring{\gamma}_{qc} + i\gamma^W_{qc}$, where $\mathring{\gamma}_{qc}$ is the (real) strong T-even amplitude. Then to first order in γ^W, and defining $(\theta_T)_{cc'} = i\theta_{cc'}$ where $\theta_{cc'}$ is antisymmetric

$$\Delta\sigma_q = \frac{2\pi}{k^2}\frac{4}{\Gamma_q}\sum_{cc'}\left[\gamma^W_{qc}\mathring{\gamma}_{qc'}\right]\theta_{cc'} \tag{3.7}$$

For the special case of two channels, one (C_1) coupled weakly and the other (C_0) coupled strongly

$$\Delta\sigma_q = \frac{2\pi}{k^2}\frac{4}{\Gamma_q}\gamma^W_{qc_1}\mathring{\gamma}_{qc_0} \tag{3.8}$$

The asymmetry, $P_q \equiv \frac{\Delta\sigma_q}{2\sigma_q}$ is then given by (ignoring the background contribution)

$$P_q = \gamma^W_{qc_1}/\mathring{\gamma}_{qc_0} \tag{3.9}$$

Generally, P_q will have a vanishing average value because of the random nature of γ^W_{qc}, and $\mathring{\gamma}_{qc_0}$. This situation changes if a local $2p-1h$ doorway dominates the TRSB mixing. Just as in the PNC case [4], whose fine structure has been recently analyzed in [17], the P_q, Eq. (3.9), will have a definite sign.

We mention here that the detailed nature of the T-violation experiment depends on the T-violating operator θ_T. Several forms may be cited. For parity non-conserving, time reversal violating, these are

$$\theta_{TP,1} = (\vec{\sigma}\cdot\hat{q}) \tag{3.10}$$

$$\theta_{TP,2} = \left(\vec{\sigma}\times\vec{I}\right)\cdot\hat{p}\,. \tag{3.11}$$

The time reversal violating P-even interactions are more complicated. We list these operators in terms of the unit vectors $\hat{q}=\hat{k'}-\hat{k}$, $\hat{p}=\hat{k}+\hat{k'}$ and $\hat{n}=\hat{p}\times\hat{q}$

$$\theta_{T,1} = i[\vec{\sigma}\times\vec{I}\cdot\hat{n}] \tag{3.12}$$

$$\theta_{T,2} = (\vec{\sigma}\times\vec{I}\cdot\hat{q})(\vec{I}\cdot\hat{q}) \tag{3.13}$$

$$\theta_{T,3} = \left(\vec{\sigma}\times\vec{I}\cdot\hat{p})(\vec{I}\cdot\hat{p}\right) \tag{3.14}$$

where $\vec{\sigma}/2$ is the spin of the nucleon, $\vec{I}$ the spin of the target nucleus. In a neutron transmission experiment where the total cross sections are measured, only $\theta_{TP,2}$ and $\theta_{T,3}$ survive. It is clear that in order to see time reversal violation both for P-odd and P-even, we must have at least two channel spins coupled by the violating interaction. As examples we mention the transition ${}^1P_1 \to {}^3S_1$ caused by the T-odd, P-odd interaction and the transition ${}^1P_1 \to {}^3P_1$ caused by a P-even, T-odd interaction. In particular, the transition ${}^1P_1 \to {}^3S_1$ mentioned above, which could occur in the neutron scattering from a spin 1/2 nucleus, is particularly interesting as it resembles the P-odd T-even case studied in Ref. [4] except for the change in channel spin. We suggest that to get a measurable P_q, Eq. (3.10), one comes in a ${}^1P_1$ state in a nucleus where the single particle P-wave strength function exhibits a minimum and comes out in a centrifugal barrier uninhibited ${}^3S_1$-state sitting at a maximum in the corresponding s-wave strength function. In the $A \simeq 180$ and 140 region one encounters such a situation [19]. The nucleus ${}^{139}La$ considered in Ref. [20] seems to be a good candidate to study TRSB.

IV. CONCLUSIONS

In conclusion, we have discussed the theory of TRSB in nuclear reactions. We first analyzed the case of overlapping resonances and showed that the cross-section correlation function C^2_{ab} depends explicitly on the channels, contrary to what has been suggested previously. We then discussed the low-energy (eV's) isolated resonances case and derived a P_q expression for the asymmetry P_q at a given resonance. A more detailed account of the present work will appear elsewhere [21].

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